

Improved SSOR-based signal detector by using based on gauss-seidel method for large-scale MIMO systems

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Abstract: As the core technology in large-scale MIMO system, the reliability of signal detection has an important impact on the whole system. Traditional linear detectors, such as the zero-forcing (ZF), often involve very complex matrix inverse calculations when the number of antennas in large-scale MIMO systems is too large. In this paper, we first introduce the traditional signal detector based on SSOR method, the iterative operation is used to replace the complex matrix inverse calculation, thus reducing the computational complexity. Then, the Gauss-Seidel method is proposed to calculate the initial iteration value, and the simulation results show that the bit error rate (BER) performance of the improved SSOR method is greatly improved.

Keywords: signal detector, low complexity, large-scale MIMO systems, Gauss-Seidel (GS), symmetric successive over-relaxation (SSOR).

1. Introduction

With the development of mobile communication in recent years, mobile terminal and mobile service also keep growing, it makes the application of multiple -input multiple-output (MIMO) technique more and more widely. such as the fourth mobile generation of communication technology (4G) and wireless local area network (LAN) standard IEEE 802.11n[1-2].Unlike the conventional small-scale MIMO systems, massive MIMO ,which equips dozens or even hundreds of antenna arrays in base station to simultaneously serves single antenna users [3] The bigger system storage space, the faster information transfer rates and the better link reliability have made the massive MIMO an important technology of 5G wireless communication [4].However ,as the number of antennas in the base station, the complexity of the algorithm in the system also increases rapidly. Therefore, it needs the algorithm which has the good detection performance and low complexity at the same time.

Common signal detection algorithms of MIMO systems can be divided into two categories [5], the linear one and the nonlinear one. The nonlinear maximum likelihood (ML) signal detector is widely regarded as the optimal signal detector [6]. However, these methods is impractical for massive MIMO because it utilizes the ergodic property to ensure optimal performance of the algorithm. For reducing the complexity, several nonlinear detectors are proposed, such as the sphere decoding (SD) signal detector [7], but it still leads to much complexity with the number of the antennas increase in base station. Therefore, people proposal the linear detector algorithm, such as the zero-forcing (ZF) signal detector or the minimum mean square error (MMSE) signal detector [8-9]. They enable the performance

to arrive at the near-perfect performance in the massive MIMO systems. Nonetheless, these signal detectors usually involve complex matrix inversion, when the number of users increase numerous, they had to entail the large number of unfavorable matrix inversion of large size. Recently, some simplified algorithms appear, such as the signal detector based on SSOR [10], these methods utilize iteration instead of matrix inversion. However, when they the operation of iteration, detection performance will become poor, because they often regard zero as the initial solution.

In this paper, we propose a more precise initial solution to improve the signal detection algorithm by using the SSOR method for massive MIMO systems. The initial solution is calculated by the based-on Gauss-Seidel method, which achieve the better detection precision and faster rate of convergence than the original basis. We conducted simulation with two different conditions that the 128 BS antennas ,32 single-antenna users and 256 BS antennas ,64 single-antenna users. Analysis shows that, with the same number of subscribers and base station antennas, signal detector based on SSOR method which utilize the proposed initial solution achieve the better performance than the traditional method. For example, when the iterations is 2 and SNR is 35, The BER performance of the improved method can be increased by one order of magnitude compared with the basic algorithm

The rest of the paper is organized as follows. Section II briefly introduces the establishment of the system model. Section III describes iterative process based on SSOR method and how to choose the appropriate initial value of iteration, in this way to improve the BER performance of the signal detector. Section IV shows the results of the simulation experiment about improved BER performance based on SSOR algorithm. Finally, the conclusions are given in Section V.

2. System Model

In order to simplify, this article assume that the user device is set as the system model of large-scale MIMO uplink with single antenna. In addition, the base station has N receiving antennas, and the number of user transmission antennas has K . There are usually $N \gg K$, such as $N=128$ and $K=16$ [10]. the signal from K users can be represented as a column vector s of size $K \times 1$. And then s is transmitted to the receiving antennas through the transmission channel matrix H , H is consistent with Rayleigh fading channel matrix, and its mean value is 0 and variance is 1. After the signal data is received by the antenna at the receiving , the signal y at the receiving end is obtained by demodulation, y is a column vector of size $N \times 1$, and it can be expressed as the following formula

$$y = Hs + n, \quad (1)$$

where n is a vector with the size of $N \times 1$, which conforms to the additive White Gaussian noise, and the variance is σ^2 and the mean value is 0.

The transmission channel matrix H can be obtained through time domain or frequency domain pilot training and other methods at BS, so we can use the signal y that the receive at the receiver to estimate the signal s that we send at the transmitter. Traditional linear detection estimation methods, such as the estimation of the ZF signal detector, can be represented as

$$\hat{s} = W^{-1}\hat{y}, \quad (2)$$

where we define $W = H^H H$, $\hat{y} = H^H y$

It can be seen the ZF signal detector can calculate the data sent by the transmitter. However, it involves the matrix inversion. It will lead to very high computational complexity, when the size of W is very large.

3. Improved SSOR based algorithm for initial values

In this section, firstly, we will introduce the iterative process based on SSOR method to solve the problem of high computational complexity which caused by matrix inversion in traditional linear detection algorithm. Secondly, we utilize the method that based on Gauss-Seidel to calculate the initial iterative value of a more suitable algorithm, which can further improve the accuracy and speed of the algorithm.

3.1 Iteration based on SSOR method

Step 1: The matrix W is decomposed as follows

$$W = D + L + L^H \quad (3)$$

where D is the diagonal matrix which is composed of diagonal elements in matrix W , L is composed of the strictly lower triangular elements of matrix W , and the L^H is the strictly upper triangular elements of matrix W .

Step 2: Calculate the relaxation parameter ω required by iteration, which can be obtained from the following equation

$$\omega^{\text{opt}} = \frac{2}{1 + \sqrt{2(1 - \rho(B_J))}} \quad (4)$$

where $\rho(B_J)$ is the spectral radius of matrix B_J of Jacobi iteration [11].

Due to the speed of SSOR based iterative algorithm is insensitive to the relaxation parameter ω [12], when the number of the massive MIMO system users K and the base station antennas N are determined, it means that K/N stays the same, the relaxation parameter can be replaced by a simple approximate calculation, as the following:

$$\bar{\omega} = \frac{2}{1 + \sqrt{2(1 - a)}} \quad (5)$$

where $a = \left(1 + \sqrt{\frac{K}{N}}\right)^2 - 1$

Step 3: Calculate the first half-iteration based on SSOR iteration method:

$$(D + \omega L^H)\hat{s}^{(i+\frac{1}{2})} = (1 - \omega)D\hat{s}^{(i)} - \omega L^H\hat{s}^{(i)} + \omega\hat{y} \quad (6)$$

Step 4: Calculate the second half-iteration based on SSOR iteration method:

$$(D + \omega L^H)\hat{s}^{(i+1)} = (1 - \omega)D\hat{s}^{(i+\frac{1}{2})} - \omega L\hat{s}^{(i+\frac{1}{2})} + \omega\hat{y} \quad (7)$$

where $i=1,2,\dots$ in both equations is the number of iterations, and the initial iteration value of this equation is $\hat{s}^{(0)}$. In general, the initial iteration is just zero [10]. It can be seen from the (6) and (7) that the iteration based on SSOR algorithm can avoid the inverse calculation of large-scale matrix W . Thus, the computational complexity of the signal detector is greatly reduced. It should be noted that W must be diagonally dominant and symmetrically positive definite which can be used in the above iteration, and in large-scale MIMO systems, these two conditions can be met [10]. However, in practice, the initial iteration value is often not zero, and we can further improve the accuracy of the algorithm by selecting a more appropriate initial iteration value.

3.2 Improved initial iteration values

By analyzing equations (6) and (7) in Section A, we can get, In practice, if the initial iteration value is calculated as 0, a relatively large error will be caused. Therefore, we can improve the performance of the algorithm by calculating a more accurate initial iteration value.

First, we can rewrite W as the following expression [14]:

$$W = G + \sigma^2 I_k \quad (8)$$

where $G=H^H H$ presents the Gram matrix.

According to the analysis of equation (3), the massive MIMO matrix can ensure that W is diagonally dominant. Besides, in the large-scale MIMO matrix, when $N \gg K$, the channel matrix H is progressively orthogonal [13]. Thus, we can get the following formula:

$$\frac{\mathbf{h}_m^H \mathbf{h}_k}{n} \rightarrow 0, m \neq k, \quad m, k = 1, 2, \dots, K \quad (9)$$

Where \mathbf{h}_m is the m th column vector of the channel matrix \mathbf{H} .

It can be inferred from the above equation that \mathbf{W}^{-1} is a diagonally dominant. Thus, we can use \mathbf{D}^{-1} approximation to replace \mathbf{W}^{-1} with a small margin of error and approximate the initial solution as:

$$\mathbf{s}^{(0)} = \mathbf{D}^{-1} \hat{\mathbf{y}} \quad (11)$$

and \mathbf{D} is a diagonal matrix, the complexity of computing \mathbf{D} minus one is very low.

Therefore, the algorithm process can be represented by the following figure:

Algorithm 1 Improved SSOR signal detector

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1: inputs:  $\mathbf{H}$ ,  $\mathbf{y}$ 
2: initialization:
3:  $\mathbf{W} = \mathbf{G} + \sigma^2 \mathbf{I}_k$ 
4:  $\mathbf{D} = \text{diag}(\text{diag}(\mathbf{W}))$ 
5:  $\mathbf{L} = \text{tril}(\mathbf{W}, -1)$ 
6:  $\hat{\mathbf{y}} = \mathbf{H}^H \mathbf{y}$ 
7: compute  $\omega$  as the (4) in Sec.III
8: Initial iteration value:
9:  $\mathbf{s}^{(0)} = \mathbf{D}^{-1} \hat{\mathbf{y}}$ 
10: Iterations:
11: for  $k=0, \dots, 2$  do
12:  $\hat{\mathbf{s}}^{(1)} = (\mathbf{D} + \omega \mathbf{L})^{-1} \times ((1 - \omega) \times \mathbf{D} \hat{\mathbf{s}}^{(0.5)} -$ 
 $\omega \mathbf{L}^H \hat{\mathbf{s}}^{(0.5)} + \omega \hat{\mathbf{y}})$ 
13:  $\hat{\mathbf{s}} = (\mathbf{D} + \omega \mathbf{L}^H)^{-1} \times ((1 - \omega) \times \mathbf{D} \hat{\mathbf{s}}^{(1)} - \omega \mathbf{L} \hat{\mathbf{s}}^{(1)}$ 
 $+ \omega \hat{\mathbf{y}})$ 
14: end for
15: outputs:  $\hat{\mathbf{s}}$ 

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4. SIMULATION RESULTS

In order to demonstrate the performance superiority of the improved method, this article carries on the simulation experiment in MATLAB platform. Firstly, we compare the BER performance of the traditional SSOR-based algorithm and ZF signal detection algorithm. After a brief analysis and comparison of the two algorithms, we compare the BER performance between the proposed improved algorithm and the traditional SSOR-based algorithm. The modulation mode is 64QAM modulation, The simulated MIMO system parameters are 256×32 and 128×16 . The transmission channel conforms to the Rayleigh fading. The abscissa SNR in the image represents the signal-to-noise ratio and the ordinate BER means the bit error rate performance, k represents the iterations.

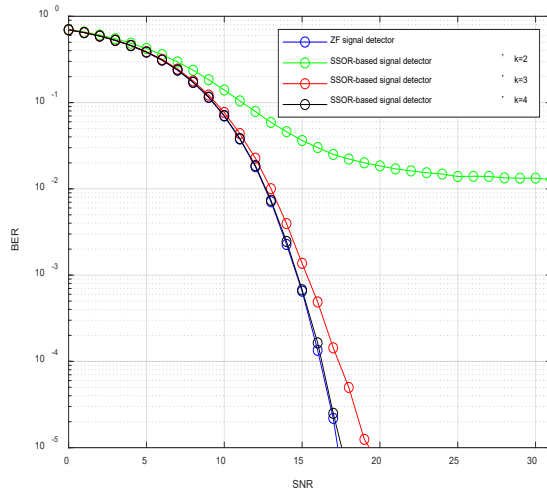


Fig.1. BER performance comparison between ZF signal and traditional SSOR-based signal detector.

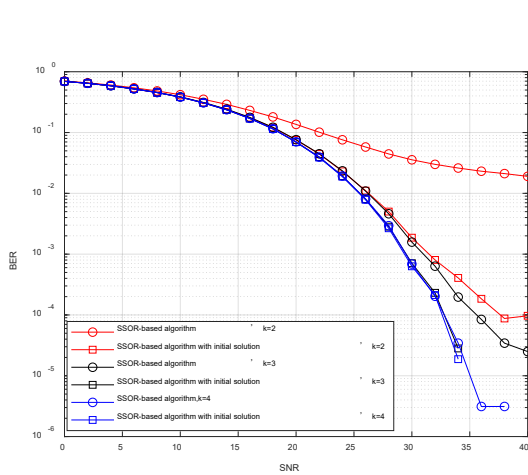


Fig.2. BER performance comparison between traditional SSOR-based method and improved algorithm. The antenna parameters of the massive MIMO system are $N \times K = 128 \times 16$.

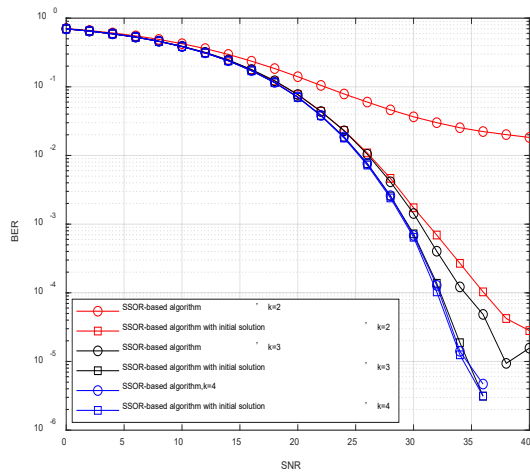


Fig.3. BER performance comparison between traditional SSOR-based method and improved algorithm. The antenna parameters of the massive MIMO system are $N \times K = 256 \times 32$.

Fig 2 and Fig 3 both compare the BER performance between the traditional SSOR-based signal detector and the proposed improved algorithm. It can be clearly seen from the Fig 2 and 3 that, the BER performance of the improved SSOR-based signal detector is obviously superior to the traditional algorithm. For example, in Fig 3, when $k=2$ and $\text{SNR}=35$, The BER performance of the improved SSOR-based signal detector can achieve 2.744×10^{-4} and the BER performance of the traditional SSOR algorithm is only 2.3×10^{-2} . The improved SSOR-based algorithm has nearly two orders of magnitude better BER performance than the traditional method, even close to the performance which the traditional method requires 3 iterations to achieve performance. At the same time, in Fig 2, when $k = 2$, in order to fulfill the BER performance of 10^{-3} , The SNR of the traditional SSOR-based method is required to be more than 40, while the SNR of the algorithm with the improved iteration initial value only needs to reach 32. This shows the improved algorithm, with the fewer iterations and smaller SNR, can achieve the BER performance of the traditional algorithm which needs higher iterations.

5. Conclusion

In this paper, we improve the signal detector based on SSOR method through the initial iteration value calculated based on Gauss-Seidel algorithm. Moreover, we verify that the improved algorithm is superior to the traditional basic algorithm in BER performance through the simulation experiment. In this paper, only the BER performance of the algorithm is improved. In the future, we can improve the bit error rate performance of the algorithm and further reduce the computational complexity of algorithms through other aspects of research.

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